Name: ______________________ **Date:** ________________ **Period:** __________

Derivatives Practice Worksheet

Instructions: Show all work clearly for each problem. Simplify answers where possible. For word problems, include units in your final answer. Use separate paper if needed.

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1. Find the derivative of $f(x) = \dfrac{x^2 \sin x}{e^x}$.

2. A particle moves along a line so that its position at time t (in seconds) is given by $s(t) = t^3 - 6t^2 + 9t + 2$ (in meters). Find the velocity of the particle at $t = 2$ seconds.

3. Find $\dfrac{dy}{dx}$ if $y = \ln\left(\sqrt{\dfrac{1+\cos x}{1-\cos x}}\right)$.

4. The radius of a sphere is increasing at a rate of 2 cm/s. How fast is the volume increasing when the radius is 5 cm? (Volume of a sphere: $V = \frac{4}{3}\pi r^3$)

5. Find the derivative of $g(x) = \arctan\left(\dfrac{x}{1+x^2}\right)$.

6. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

7. Find $f'(x)$ if $f(x) = x^{\sin x}$ for $x > 0$.

8. A conical tank (with vertex down) has a radius of 3 m at the top and a height of 6 m. Water is being pumped into the tank at a rate of 2 m^3/min. How fast is the water level rising when the water is 4 m deep? (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)

9. Find the derivative of $h(x) = \sec^{-1}(e^{2x})$.

10. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

Answer Key

1. $f'(x) = \dfrac{2x\sin x + x^2\cos x - x^2\sin x}{e^x}$ (using quotient rule: $f'(x) = \frac{(2x\sin x + x^2\cos x)e^x - x^2\sin x \cdot e^x}{e^{2x}} = \frac{2x\sin x + x^2\cos x - x^2\sin x}{e^x}$)

2. $v(t) = s'(t) = 3t^2 - 12t + 9$. At $t = 2$, $v(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3$ m/s (moving downward).

3. Simplify: $y = \frac{1}{2}\ln\left(\frac{1+\cos x}{1-\cos x}\right) = \frac{1}{2}[\ln(1+\cos x) - \ln(1-\cos x)]$. Then $y' = \frac{1}{2}\left(\frac{-\sin x}{1+\cos x} - \frac{\sin x}{1-\cos x}\right) = \frac{1}{2}\left(\frac{-\sin x(1-\cos x) - \sin x(1+\cos x)}{1-\cos^2 x}\right) = \frac{1}{2}\left(\frac{-2\sin x}{\sin^2 x}\right) = -\csc x$.

4. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Given $\frac{dr}{dt} = 2$ cm/s, $r = 5$: $\frac{dV}{dt} = 4\pi(25)(2) = 200\pi$ cm^3/s.

5. $g'(x) = \frac{1}{1+\left(\frac{x}{1+x^2}\right)^2} \cdot \frac{(1)(1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1}{1+\frac{x^2}{(1+x^2)^2}} \cdot \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1}{\frac{(1+x^2)^2+x^2}{(1+x^2)^2}} \cdot \frac{1-x^2}{(1+x^2)^2} = \frac{(1+x^2)^2}{(1+x^2)^2+x^2} \cdot \frac{1-x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2+x^2}$.

6. Let x be distance from wall to bottom, y be height of top on wall. $x^2 + y^2 = 100$. Differentiate: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$. When $x = 6$, $y = 8$, $\frac{dx}{dt} = 1$: $\frac{dy}{dt} = -\frac{6}{8}(1) = -\frac{3}{4}$ ft/s (sliding down at 0.75 ft/s).

7. $f(x) = e^{\sin x \ln x}$. Then $f'(x) = e^{\sin x \ln x}\left(\cos x \ln x + \frac{\sin x}{x}\right) = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$.

8. By similar triangles, $\frac{r}{h} = \frac{3}{6} = \frac{1}{2}$, so $r = \frac{h}{2}$. Then $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$. Differentiate: $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$. Given $\frac{dV}{dt} = 2$, $h = 4$: $2 = \frac{\pi}{4}(16)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{4\pi} = \frac{1}{2\pi}$ m/min.

9. $h'(x) = \frac{1}{|e^{2x}|\sqrt{(e^{2x})^2-1}} \cdot 2e^{2x} = \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}} = \frac{2}{\sqrt{e^{4x}-1}}$.

10. Let x be horizontal distance from station to plane, s be distance from station to plane. $s^2 = x^2 + 1^2$. Differentiate: $2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt}$. When $s = 2$, $x = \sqrt{4-1} = \sqrt{3}$, $\frac{dx}{dt} = 500$: $\frac{ds}{dt} = \frac{\sqrt{3}}{2}(500) = 250\sqrt{3}$ mi/h.